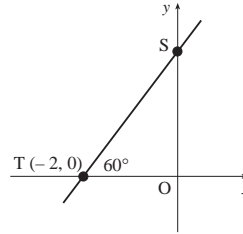


- 1 Find the equation of the line ST, where T is the point  $(-2, 0)$  and angle STO is  $60^\circ$ .

3



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
1		3	C	G2, G3	NC	05/6

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> ss use  $m = \tan \theta$
- <sup>2</sup> pd use exact value
- <sup>3</sup> ic interpret result

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $m = \tan(60^\circ)$  stated or implied by •<sup>2</sup>
- <sup>2</sup>  $m = \sqrt{3}$
- <sup>3</sup>  $y - 0 = \sqrt{3}(x - (-2))$

3 marks

### Notes

- 1 A candidate who states  $m = \tan(\theta^\circ)$ , and does not go on to use it earns no marks.

#### Incompletion 1

$$m = \tan(60^\circ)$$

$$y - 0 = \tan(60^\circ)(x - (-2))$$

- <sup>1</sup>  $\times \checkmark$
- <sup>2</sup>  $\times$
- <sup>3</sup>  $\times \checkmark$

*award 2 marks*

#### Common Error 1

$$m = \sin(60^\circ)$$

$$y - 0 = \frac{\sqrt{3}}{2}(x - (-2))$$

- <sup>1</sup>  $\times$
- <sup>2</sup>  $\times \checkmark$
- <sup>3</sup>  $\times \checkmark$

*award 2 marks*

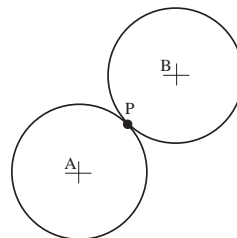
#### Alternative Method 1

- <sup>1</sup>  $OS = 2 \tan(60^\circ) = 2\sqrt{3}$
- <sup>2</sup>  $m = \frac{2\sqrt{3}}{2} = \sqrt{3}$   
(cf  $y = mx + c$ )
- <sup>3</sup>  $y = \sqrt{3}x + 2\sqrt{3}$

#### Alternative Method 2

- <sup>1</sup>  $\cos(60^\circ) = \frac{2}{ST}$  leading to  
 $ST = 4$  and  $OS = \sqrt{12}$
- <sup>2</sup>  $m = \frac{\sqrt{12}}{2}$
- <sup>3</sup>  $y - 0 = \frac{\sqrt{12}}{2}(x - (-2))$

- 2 Two congruent circles, with centres A and B, touch at P.  
Relative to suitable axes, their equations are  
 $x^2 + y^2 + 6x + 4y - 12 = 0$  and  $x^2 + y^2 - 6x - 12y + 20 = 0$ .
- (a) Find the coordinates of P.  
(b) Find the length of AB.



3  
2

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
2	a	3	C	G9, G6	CN	05/18
	b	2	C	G9	CN	

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> ic interpret equ. of circle
- <sup>2</sup> ic interpret equ. of circle
- <sup>3</sup> pd process midpoint
- <sup>4</sup> ss know how to find length
- <sup>5</sup> pd process

**Primary Method : Give 1 mark for each •**

- <sup>1</sup> centre A = (-3, -2)      | [Note 1]
- <sup>2</sup> centre B = (3, 6)
- <sup>3</sup> P = (0, 2)      **3 marks**
- <sup>4</sup>  $AB^2 = (3 - (-3))^2 + (6 - (-2))^2$  [CE 1]
- <sup>5</sup> AB = 10      [Note 2]      **2 marks**

**Notes**

- 1 at •1, •2  
Each of the following may be awarded 1 mark from the first two marks
- $A = (6, 4)$  and  $B = (-6, -12)$   
 $A = (-6, -4)$  and  $B = (6, 12)$   
 $A = (3, 2)$  and  $B = (-3, -6)$
- 2 At •5 stage, some errors lead to unsimplified surds. **DO NOT** accept unsimplified square roots of perfect squares (up to 100).  
e.g.  $\sqrt{100}$  would not gain •5.

**Alternative Method 1 for marks 1,2,3**

$$p = \frac{1}{2}(b + a)$$

- <sup>1</sup>  $b = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$
- <sup>2</sup>  $a = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$
- <sup>3</sup> P = (0, 2)      [Note 1]

**Notes**

- 1 Treat  $P = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$  as bad form.

**Alternative Method 2 for marks 4,5**

- <sup>4</sup>  $r^2 = 3^2 + 2^2 - (-12)$   
or  $r^2 = (-3)^2 + (-6)^2 - 20$
- <sup>5</sup> AB = 2r = 10

**Alternative Method 3 for marks 4,5**

- <sup>4</sup>  $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$
- <sup>5</sup> AB = 10

**Common Error 1 for (b)**

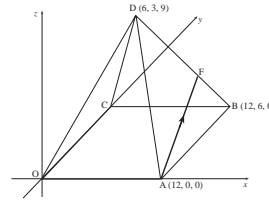
$$AB^2 = (3 + (-3))^2 + (6 + (-2))^2$$

$$AB = 4$$

- <sup>4</sup> ×
- <sup>5</sup> ×√

award 1 mark for (b)

- 3 D,OABC is a pyramid. A is the point (12, 0, 0), B is (12, 6, 0) and D is (6, 3, 9).  
 F divides DB in the ratio 2 : 1.
- (a) Find the coordinates of the point F.
- (b) Express  $\vec{AF}$  in component form.



4  
1

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
3	a	4	C	G25	CN	05/24
	b	1	C	G17	CN	

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> ss know to find  $\vec{DB}$
- <sup>2</sup> ic interpret ratio
- <sup>3</sup> pd process scalar times vector
- <sup>4</sup> ic interpret vector and end points
- <sup>5</sup> ic interpret coordinates to vector

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $\vec{DB} = \begin{pmatrix} 12-6 \\ 6-3 \\ 0-9 \end{pmatrix}$
- <sup>2</sup>  $\vec{DF} = \frac{2}{3}\vec{DB}$
- <sup>3</sup>  $\vec{DF} = \frac{2}{3} \begin{pmatrix} 6 \\ 3 \\ -9 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix}$
- <sup>4</sup> D = (6, 3, 9) so F = (10, 5, 3) **[Note 1]** 4 marks
- <sup>5</sup>  $\vec{AF} = \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$  1 mark

Notes

- 1 Do not penalise candidates who write the coordinates of F as a column vector (treat as bad form).
- 2 A correct answer to (a) with no working may be awarded one mark only.
- 3 For guessing the coordinates of F, no marks should be awarded in (a).  
 1 mark is still available in (b) provided the guess in (a) is geographically compatible with the diagram  
 ie  $0 \leq x \leq 12$   
 $3 \leq y \leq 6$   
 $0 \leq z \leq 9$
- 4 In (a)  
 Where the ratio has been reversed (ie 1:2) leading to F=(8, 4, 6) then 3 marks may be awarded (•<sup>1</sup>, •<sup>3</sup>, •<sup>4</sup>).
- 5 In (b)  
 Accept  $\vec{AF} = -2\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$  for •<sup>5</sup>.

**Alternative Method 1 [Marks 1-4]**

- <sup>1</sup>  $\vec{DF} = 2\vec{FB}$  s/i by •<sup>2</sup>
- <sup>2</sup>  $f - d = 2b - 2f$
- <sup>3</sup>  $3f = 2 \begin{pmatrix} 12 \\ 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix}$
- <sup>4</sup> F = (10, 5, 3) **[Note 1]**

**Alternative Method 3 [Marks 1-5]**

- <sup>1</sup>  $\vec{AF} = \vec{AB} + \vec{BF}$
- <sup>2</sup>  $\vec{AF} = \vec{AB} + \frac{1}{3}\vec{BD}$
- <sup>3</sup>  $\vec{AF} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} + \frac{1}{3} \left[ \begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix} - \begin{pmatrix} 12 \\ 6 \\ 0 \end{pmatrix} \right]$
- <sup>4</sup>  $\vec{AF} = \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$
- <sup>5</sup> (A = (12, 0, 0 so) F = (10, 5, 3)

**Alternative Method 2 [Marks 1-4]**

- <sup>1</sup>  $f = \frac{mb + nd}{m + n}$  s/i by •<sup>3</sup>
- <sup>2</sup>  $m = 2, n = 1$  s/i by •<sup>3</sup>
- <sup>3</sup>  $f = \frac{1}{3} \left[ 2 \begin{pmatrix} 12 \\ 6 \\ 0 \end{pmatrix} + 1 \begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix} \right]$
- <sup>4</sup> F = (10, 5, 3) **[Note 1]**

**Alternative Method 4 [Marks 1-4]**

x	6	.	10	12	• <sup>1</sup>
y	3	.	5	6	• <sup>2</sup>
z	9	.	3	0	• <sup>3</sup>
so F = (10, 5, 3)					• <sup>4</sup>

- 4 Functions  $f(x) = 3x - 1$  and  $g(x) = x^2 + 7$  are defined on the set of real numbers.
- (a) Find  $h(x)$  where  $h(x) = g(f(x))$ . 2
- (b) (i) Write down the coordinates of the minimum turning point of  $y = h(x)$ .
- (ii) Hence state the range of the function  $h$ . 2

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
4	a	2	C	A4	NC	05/7
	b	2	C	A1	NC	

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> ic interpret comp. function build-up
- <sup>2</sup> ic interpret comp. function build-up
- <sup>3</sup> ic interpret function
- <sup>4</sup> ic interpret function

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $g(3x - 1)$  *stated or implied by •2*
- <sup>2</sup>  $(3x - 1)^2 + 7$  2 marks
- <sup>3</sup>  $\left(\frac{1}{3}, 7\right)$  *[Note 1]*
- <sup>4</sup>  $y \geq 7$  *[Note 2]* 2 marks

**Notes**

1 For •3

No justification is required for •3. Candidates may choose to differentiate etc but may still only earn one mark for a correct answer.

2 For •4

Accept  $y > 7, h \geq 7, h > 7, h(x) > 7, h(x) \geq 7$   
Do not accept  $x \geq 7, x > 7$

**Common Error No.1**

- <sup>1</sup>  $\times f(x^2 + 7)$
  - <sup>2</sup>  $\times \sqrt{3x^2 + 20}$
  - <sup>3</sup>  $\times \sqrt{(0, 20)}$
  - <sup>4</sup>  $\times \sqrt{y \geq 20}$
- award 3 marks*

Notes 1 & 2 apply.

5 Differentiate  $(1 + 2 \sin(x))^4$  with respect to  $x$ .

2

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
5		2	A	C20, C21	CN	05/28

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> pd start differentiation process
- <sup>2</sup> pd use the chain rule

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $4(1 + 2 \sin(x))^3$
- <sup>2</sup>  $\dots \times 2 \cos(x)$

2 marks

**Common Error 1**

- <sup>1</sup>  $\times 1 + 2 \sin^4(x)$
  - <sup>2</sup>  $\times \sqrt{8 \sin^3(x) \times \cos(x)}$
- award 1 mark*

**Common Error 2**

- <sup>1</sup>  $\times 1 + 16 \sin^4(x)$
  - <sup>2</sup>  $\times \sqrt{64 \sin^3(x) \times \cos(x)}$
- award 1 mark*

**Common Error 3**

[mixture of differentiating and integrating]

- <sup>1</sup>  $\times \frac{1}{4}(1 + 2 \sin(x))^3$
  - <sup>2</sup>  $\times \frac{1}{2} \cos(x)$
- award 0 marks*

**Common Error 4**

- <sup>1</sup>  $\times 4(1 + 2 \sin(x))^5$
  - <sup>2</sup>  $\times \sqrt{\times 2 \cos(x)}$
- award 1 mark*

- 6 (a) The terms of a sequence satisfy  $u_{n+1} = ku_n + 5$ . Find the value of  $k$  which produces a sequence with a limit of 4. 2
- (b) A sequence satisfies the recurrence relation  $u_{n+1} = mu_n + 5$ ,  $u_0 = 3$ .
- (i) Express  $u_1$  and  $u_2$  in terms of  $m$ .
- (ii) Given that  $u_2 = 7$ , find the value of  $m$  which produces a sequence with no limit. 5

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
6	a	2	C	A13	CN	05/42
	b	5	B	A11, A13	CN	

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> ss know how to find limit
- <sup>2</sup> pd process
- <sup>3</sup> ic interpret rec. relation
- <sup>4</sup> ic interpret rec. relation
- <sup>5</sup> pd arrange in standard form
- <sup>6</sup> pd process a quadratic
- <sup>7</sup> ic use limit condition

**Primary Method : Give 1 mark for each •**

- <sup>1</sup> e.g.  $4 = k \times 4 + 5$  [Notes 1,2,3]
- <sup>2</sup>  $k = -\frac{1}{4}$  2 marks
- <sup>3</sup>  $u_1 = 3m + 5$
- <sup>4</sup>  $u_2 = m(3m + 5) + 5$  [Note 4]  
 $(m(3m + 5) + 5 = 7)$
- <sup>5</sup>  $3m^2 + 5m - 2 = 0$  [Note 5]
- <sup>6</sup>  $(3m - 1)(m + 2) = 0$
- <sup>7</sup>  $m = -2$  5 marks

**Alternative Method 1 for (a)**

$$\text{Using } L = \frac{b}{1-a}$$

$$\bullet^1 \quad 4 = \frac{5}{1-k}$$

$$\bullet^2 \quad k = -\frac{1}{4}$$

**Alternative Method 2 for (a)**

$$L = kL + 5$$

$$kL = L - 5$$

$$\bullet^1 \quad k = \frac{L-5}{L}$$

$$\bullet^2 \quad k = \frac{4-5}{4} = -\frac{1}{4}$$

#### Notes

##### for (a)

- 1 Guess and Check  
 Guessing  $k = -0.25$  and checking algebraically or iteratively that this does yield a limit of 4 may be awarded 1 mark.
- 2 No working  
 Simply stating that  $k = -0.25$  earns no marks.
- 3 Wrong formula  
 Work using an incorrect 'formula' leading to a valid value of  $k$  (ie  $|k| < 1$ ) may be awarded 1 mark.

##### for (b)

- 4 If  $u_2$  is not a quadratic, then no further marks are available.
- 5 An "=0" must appear at least once in working at the •5/•6 stage.
- 6 For candidates who make errors leading to no values outside the range  $-1 < m < 1$ , or to two values outside the range, then they must say why they are accepting or rejecting in order to gain •7
- 7 For •7, either crossing out the "1/3" or underlining the "-2" is the absolute minimum communication required for this i/c mark. [A statement would be preferable]

#### Common Error 1

$$\bullet^1 \quad \times \quad 4 = \frac{5}{1-a}$$

$$\bullet^2 \quad \times \quad \sqrt{a} = -\frac{1}{4}$$

award 1 mark

#### Common Error 2

$$\bullet^3 \quad \sqrt{u_1 = 3m + 5}$$

$$\bullet^4 \quad \times \quad u_2 = 3m^2 + 5$$

$$\bullet^5 \quad \times \quad 3m^2 = 2 \quad \text{or equivalent}$$

$$\bullet^6 \quad \times \quad m = \sqrt{\frac{2}{3}} \quad (\text{eased})$$

$$\bullet^7 \quad \times \quad \sqrt{\quad} \text{ there are no values which do not yield a limit}$$

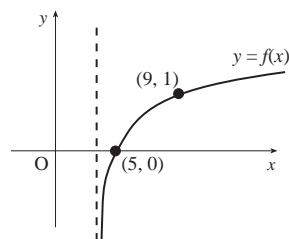
award 2 marks

7 The function  $f$  is of the form  $f(x) = \log_b(x - a)$ .

The graph of  $y = f(x)$  is shown in the diagram.

(a) Write down the values of  $a$  and  $b$ .

(b) State the domain of  $f$ .



2  
1

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
7	a	2	C	A7	NC	05/9
	b	1	C	A1	NC	

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> ic interpret the translation
- <sup>2</sup> ic interpret the base
- <sup>3</sup> ic interpret diagram

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $a = 4$                       |                      [Note 1]                      2 marks
- <sup>2</sup>  $b = 5$
- <sup>3</sup> domain is  $x > a$                       [Note 2]                      1 mark

#### Notes

- 1 No justification is required for marks 1 and 2. BUT simply stating

$$0 = \log_b(5 - a) \text{ and } 1 = \log_b(9 - a)$$

with no further work earns no marks.

However

$$1 = \log_b(9 - a) \text{ and } b = 9 - a$$

may be awarded 1 mark.

Of course to gain the other mark, both values would need to be stated.

- 2 Clearly  $x > 4$  is correct

but **do not** accept a domain of  $x \geq 4$ .

- 8 A function  $f$  is defined by the formula  $f(x) = 2x^3 - 7x^2 + 9$  where  $x$  is a real number.
- (a) Show that  $(x - 3)$  is a factor of  $f(x)$ , and hence factorise  $f(x)$  fully. 5
- (b) Find the coordinates of the points where the curve with equation  $y = f(x)$  crosses the  $x$ - and  $y$ -axes. 2
- (c) Find the greatest and least values of  $f$  in the interval  $-2 \leq x \leq 2$ . 5

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
8	a	5	C	A21	NC	05/10
	b	2	C	A21	NC	
	c	5	B	C11	NC	

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> ss know to use  $x = 3$
- <sup>2</sup> pd complete strategy
- <sup>3</sup> ic interpret zero remainder
- <sup>4</sup> ic interpret quadratic factor
- <sup>5</sup> pd complete factorising

**Primary Method : Give 1 mark for each •**

- <sup>1</sup> eg 3 

2	-7	0	9
---	----	---	---
- <sup>2</sup> eg 3 

2	-7	0	9
	6	-3	-9
2	-1	-3	0
- <sup>3</sup> remainder is zero so  $(x - 3)$  is a factor **[Note 1]**
- <sup>4</sup>  $2x^2 - x - 3$
- <sup>5</sup>  $(x - 3)(2x - 3)(x + 1)$  **stated explicitly** 5 marks

**Notes**

In the Primary method, (a)

- 1 Candidates must show some acknowledgement of the result of the synthetic division. Although a statement w.r.t. the zero is preferable, accept something as simple as "underlining" the zero.
- 2 Candidates may use a second synthetic division to complete the factorisation. •4 and •5 are available.

**Alternative method 1 (marks 1-5) (linear factor by substitution)**

- <sup>1</sup>  $f(3) = \dots$
- <sup>2</sup>  $f(3) = 2 \times 3^3 - 7 \times 3^2 + 9 = 54 - 63 + 9 = 0$
- <sup>3</sup> eg 3 

2	-7	0	9
	6		
2	-1	-3	0
- <sup>4</sup>  $2x^2 - x - 3$
- <sup>5</sup>  $(x - 3)(2x - 3)(x + 1)$

**Alternative method 3 (marks 1-5) (quad factor by inspection)**

- <sup>1</sup>  $f(3) = \dots$
- <sup>2</sup>  $f(3) = 2 \times 3^3 - 7 \times 3^2 + 9 = 54 - 63 + 9 = 0$
- <sup>3</sup>  $(x - 3)(2x^2 \dots\dots\dots)$
- <sup>4</sup>  $(x - 3)(2x^2 - x - 3)$
- <sup>5</sup>  $(x - 3)(2x - 3)(x + 1)$

**Alternative method 2 (marks 1-5) (long division)**

- <sup>1</sup>  $x - 3 \overline{) 2x^3 - 7x^2 + 9}$
- <sup>2</sup>  $x - 3 \overline{) 2x^3 - 7x^2 + 9}$
- <sup>3</sup> remainder is zero so  $(x - 3)$  is a factor
- <sup>4</sup>  $(x - 3)(2x^2 - x - 3)$
- <sup>5</sup>  $(x - 3)(2x - 3)(x + 1)$



- 8 A function  $f$  is defined by the formula  $f(x) = 2x^3 - 7x^2 + 9$  where  $x$  is a real number.
- (a) Show that  $(x-3)$  is a factor of  $f(x)$ , and hence factorise  $f(x)$  fully. 5
- (b) Find the coordinates of the points where the curve with equation  $y = f(x)$  crosses the  $x$ - and  $y$ -axes. 2
- (c) Find the greatest and least values of  $f$  in the interval  $-2 \leq x \leq 2$ . 5

- <sup>6</sup> ic interpret  $y$ -intercept
- <sup>7</sup> ic interpret  $x$ -intercepts
- <sup>8</sup> ss set derivative to zero
- <sup>9</sup> pd solve
- <sup>10</sup> ss evaluate function at an end point
- <sup>11</sup> ic interpret results
- <sup>12</sup> ic interpret results

**Primary Method : Give 1 mark for each •**

- <sup>6</sup> (0,9) [Note 3]
  - <sup>7</sup>  $(-1,0), (\frac{3}{2},0), (3,0)$  2 marks
  - <sup>8</sup>  $6x^2 - 14x = 0$
  - <sup>9</sup>  $x = 0$  or  $x = \frac{14}{6}$  [Note 6]
  - <sup>10</sup>  $f(-2) = -35$  **OR**  $f(2) = -3$
  - <sup>11</sup> greatest value = 9
  - <sup>12</sup> least value = -35 [Note 7]
- 5 marks

**Notes**

In the Primary method (b)

- 3 Only coordinates are acceptable for full marks. Simply stating the values at which it cuts the  $x$ - and  $y$ -axes may be awarded 1 mark (out of 2).
- 4 If all the coordinates are "round the wrong way" award 1 mark.
- 5 If the brackets are missing, treat as bad form.

**In the Primary method (c)**

- 6 Ignore any attempt to evaluate function at  $x = 7/3$ .
- 7 •<sup>11</sup> and •<sup>12</sup> are not available unless both end points and the st. points have been considered.

**In the Alt.5 method (c)**

- 8 •<sup>12</sup> is not available unless both end points have been considered.

**In (c)**

- 9 Some candidates simply draw up a table using integer values from  $-2$  to  $2$  and make conclusions from it. This earns •<sup>9</sup> (Primary) ONLY, provided that one of the end points is correct.

**Alternative method 5 (marks 8-12) (nature table)**

- <sup>8</sup>  $6x^2 - 14x = 0$
- <sup>9</sup>  $x = 0$  or  $x = \frac{14}{6}$  [Note 6]
- <sup>10</sup> nature table showing  $x = 0$  is max. tp and the greatest (maximum) value is 9
- <sup>11</sup>  $f(-2) = -35$  **OR**  $f(2) = -3$
- <sup>12</sup> least value = -35 [Note 8]

- 9 If  $\cos(2x) = \frac{7}{25}$  and  $0 < x < \frac{\pi}{2}$ , find the exact values of  $\cos(x)$  and  $\sin(x)$ .

4

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
9		4	C	T8	NC	05/16

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> ss use double angle formula
- <sup>2</sup> pd process
- <sup>3</sup> pd process
- <sup>4</sup> pd process

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $2\cos^2(x) - 1 = \frac{7}{25}$
- <sup>2</sup>  $\cos^2(x) = \frac{32}{50}$
- <sup>3</sup>  $\cos(x) = \frac{4}{5}$
- <sup>4</sup>  $\sin(x) = \frac{3}{5}$

4 marks

**Notes**

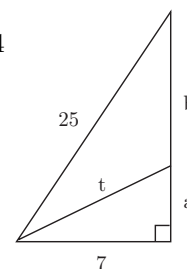
- 1 In the event of  $\cos^2(x) - \sin^2(x)$  being used, no marks are available until the equation reduces to a quadratic in either  $\cos(x)$  or  $\sin(x)$ .
- 2  $\cos(x) = \pm \frac{4}{5}$ ,  $\sin(x) = \pm \frac{3}{5}$  loses •3.
- 3 •3 and •4 are only available as a consequence of attempting to apply the double angle formula. (This note does not apply to alt. method 2)
- 4 Guess and Check.  
For guessing that  $\cos(x) = \frac{4}{5}$  and  $\sin(x) = \frac{3}{5}$ , substituting them into any valid expression for  $\cos(2x)$  and getting  $7/25$ , award 1 mark only.

**Alternative Method 1**

- <sup>1</sup>  $1 - 2\sin^2(x) = \frac{7}{25}$
- <sup>2</sup>  $\sin^2(x) = \frac{18}{50}$
- <sup>3</sup>  $\sin(x) = \frac{3}{5}$
- <sup>4</sup>  $\cos(x) = \frac{4}{5}$

**Alternative Method 2**

- <sup>1</sup>  $(7, 24, 25)$  triangle  $\Rightarrow a + b = 24$   
*and* angle bisector  $\Rightarrow \frac{a}{b} = \frac{7}{25}$
- <sup>2</sup>  $a + \frac{25}{7}a = 24 \Rightarrow a = \frac{21}{4}$
- <sup>3</sup>  $(21, 28, 35)$  triangle  $\Rightarrow t = \frac{35}{4}$
- <sup>4</sup>  $\cos(x) = \frac{4}{5}$  *and*  $\sin(x) = \frac{3}{5}$



**Common Error 1**

$$2\cos^2(x) - 1 = \frac{7}{25}$$

$$\cos^2(x) = \frac{64}{25}$$

$$\cos(x) = \frac{8}{5}$$

$$\sin(x) = \frac{6}{5}$$

- <sup>1</sup> ✓
- <sup>2</sup> ×
- <sup>3</sup> ×
- <sup>4</sup> ×

*award 1 mark only*

**Common Incompletion 1**

- <sup>1</sup> ✓  $2\cos^2(x) - 1 = \frac{7}{25}$

- <sup>2</sup> ✓  $\cos^2(x) = \frac{32}{50}$

- <sup>3</sup> ×  $\cos(x) = \sqrt{\frac{32}{50}}$

- <sup>4</sup> ×  $\sin(x) = \sqrt{\frac{18}{50}}$

*award 3 marks*

- 10 (a) Express  $\sin(x) - \sqrt{3}\cos(x)$  in the form  $k\sin(x - a)$  where  $k > 0$  and  $0 \leq a \leq 2\pi$ . 4
- (b) Hence, or otherwise, sketch the curve with equation  $y = 3 + \sin(x) - \sqrt{3}\cos(x)$  in the interval  $0 \leq x \leq 2\pi$ . 5

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
10	a	4	C	T13	NC	05/27
	b	5	A	T15	NC	

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> ic expand
- <sup>2</sup> ic compare coefficients
- <sup>3</sup> pd process  $k$
- <sup>4</sup> pd process angle
- <sup>5</sup> ic state equation
- <sup>6</sup> ic completing graph
- <sup>7</sup> ic completing graph
- <sup>8</sup> ic completing graph
- <sup>9</sup> ic completing graph

#### Notes

##### In the whole question

Do not penalise more than once for not using radians.

##### In (a)

- 1  $k(\sin(x)\cos(a) - \cos(x)\sin(a))$  is acceptable for •1
- 2 No justification is required for •3
- 3 •<sup>3</sup> is not available for an unsimplified  $\sqrt{4}$
- 4  $2(\sin(x)\cos(a) - \cos(x)\sin(a))$  is acceptable for •1 and •3 or  $2\sin(x)\cos(a) - 2\cos(x)\sin(a)$
- 5 Candidates may use any form of the wave equation to start with as long as their final answer is in the form  $k\sin(x - a)$ . If it is not, then •<sup>4</sup> is not available.
- 6 •<sup>4</sup> is only available for an answer in radians.
- 7 Treat  $k\sin(x)\cos(a) - \cos(x)\sin(a)$  as bad form only if •2 is gained.

##### In (b)

- 8 The **correct** sketch need not include annotation of max, min or intercept for •5 to be awarded but you would need to see the graph lying between  $y = 1$  and  $y = 5$ .
- 9 •<sup>6</sup> is available for one cycle of any sinusoidal curve of period  $2\pi$  except  $y = \sin(x)$ . Some evidence of a scale is required.
- 10 For •<sup>7</sup>, accept 1.3 in lieu of  $3 - \sqrt{3}$
- 11 Do not penalise graphs which go beyond the interval  $0 \dots 2\pi$ .

#### Primary Method : Give 1 mark for each •

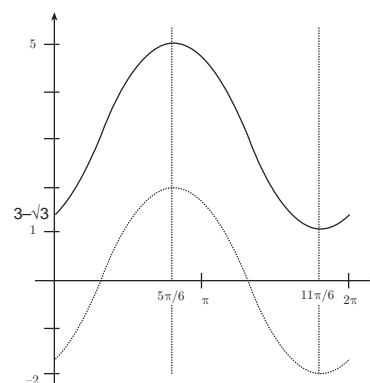
- <sup>1</sup>  $k\sin(x)\cos(a) - k\cos(x)\sin(a)$  | STATED EXPLICITLY
- <sup>2</sup>  $k\cos(a) = 1, k\sin(a) = \sqrt{3}$  | STATED EXPLICITLY
- <sup>3</sup>  $k = 2$  | [Notes 1-7]
- <sup>4</sup>  $a = \frac{\pi}{3}$  | 4 marks
- <sup>5</sup>  $y = 3 + 2\sin\left(x - \frac{\pi}{3}\right)$  | stated or implied by a correct sketch [Note 8]
- a sketch showing | [Notes 9,10]
- <sup>6</sup> a sinusoidal curve
- <sup>7</sup>  $y$ -intercept at  $(0, 3 - \sqrt{3})$  and no  $x$ -intercepts
- <sup>8</sup> max at  $\left(\frac{5\pi}{6}, 5\right)$  | 5 marks
- <sup>9</sup> min at  $\left(\frac{11\pi}{6}, 1\right)$

#### Alternative marking for •8 and •9

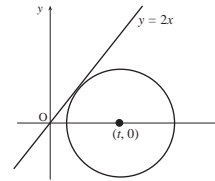
- <sup>8</sup> max at  $x = \frac{5\pi}{6}$  **and** min at  $x = \frac{11\pi}{6}$
- <sup>9</sup> graph lies between  $y = 1$  and  $y = 5$

#### Alternative method for •5 to •9 (Calculus)

- <sup>5</sup>  $\frac{dy}{dx} = \cos(x) + \sqrt{3}\sin(x) = 0$
- <sup>6</sup>  $\tan(x) = -\frac{1}{\sqrt{3}}$
- <sup>7</sup> max at  $\left(\frac{5\pi}{6}, 5\right)$
- <sup>8</sup> min at  $\left(\frac{11\pi}{6}, 1\right)$
- <sup>9</sup>  $x = 0 \Rightarrow y = 3 - \sqrt{3}$   
**and annotated sketch.**



- 11 (a) A circle has centre  $(t, 0)$ ,  $t > 0$ , and radius 2 units.  
Write down the equation of the circle.
- (b) Find the exact value of  $t$  such that the line  $y = 2x$  is a tangent to the circle.



1  
5

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
11	a	1	C	G10	CN	05/28
	b	4	A	G13	CN	

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> ic state equ. of circle
- <sup>2</sup> ss substitute
- <sup>3</sup> pd rearrange in standard form.
- <sup>4</sup> ss know to use "discriminant = 0"
- <sup>5</sup> ic identify "a", "b" and "c"
- <sup>6</sup> pd process

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $(x - t)^2 + (y - 0)^2 = 2^2$  1 mark
- <sup>2</sup>  $(x - t)^2 + (2x)^2 = 4$
- <sup>3</sup>  $5x^2 - 2tx + t^2 - 4 = 0$
- <sup>4</sup> " $b^2 - 4ac$ " = 0 [Note 1]
- <sup>5</sup>  $a = 5, b = -2t, c = t^2 - 4$
- <sup>6</sup>  $4t^2 - 20(t^2 - 4) = 0$
- and**  $t = \sqrt{5}$  [Note 2] 5 marks

**Notes**

- 1 Subsequent to trying to use an expression masquerading as the discriminant e.g.  $a^2 - 4bc = 0$ , **only •5** (from the last two marks) is still available.
- 2 Treat  $t = \pm\sqrt{5}$  as bad form.

**Common Error No. 1**

- <sup>5</sup>  $\times a = 5, b = -2, c = t^2 - 4$
- <sup>6</sup>  $4 - 20(t^2 - 4) = 0$
- $20t^2 = 84$
- $\times \sqrt{t} = \sqrt{\frac{21}{5}} \text{ or } \sqrt{4.2}$

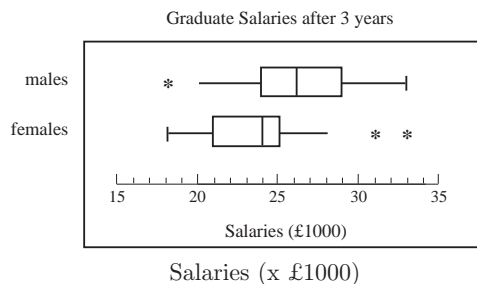
**Alternative Method 1 (for (b))**

- Let  $P$  be point of contact,  $C$  the centre of the circle.  
Consider triangle  $OPC$ .
- $OPC = 90^\circ$  (tgt/radius)
  - $PC = 2$  (radius)
  - $CP/OP = \tan(COP) = 2$  (gradient of tgt)
  - Hence  $OP = 1$
  - and, by Pythagoras,  $t = OC = \sqrt{(2^2 + 1^2)} = \sqrt{5}$ .

**Alternative Method 2 (for (b))**

- $y = 2x \Rightarrow m_{tgt} = 2$  and  $m_{rad} = -\frac{1}{2}$
- <sup>2</sup> equ of radius is  $x + 2y = t$   
ie  $x - t = -2y$
  - <sup>3</sup>  $(-2y)^2 + y^2 = 4$
  - <sup>4</sup>  $y = \frac{2}{\sqrt{5}}$
  - <sup>5</sup>  $x = \frac{1}{2}y \Rightarrow x = \frac{1}{\sqrt{5}}$
  - <sup>6</sup>  $t = x + 2y \Rightarrow t = \sqrt{5}$

S1 The boxplot shows the salaries of male and female graduates working for a large company at the end of their third year of employment.  
[3] Compare the salaries of these males and females.



3

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
S1		3	C	4.1.3/4	NC	05/70

- <sup>1</sup> ic comment
- <sup>2</sup> ic comment
- <sup>3</sup> ic comment

- <sup>1</sup> one comment from list
  - <sup>2</sup> one comment from list
  - <sup>3</sup> one comment from list 3 marks
- males had higher salaries on average by £2000  
range of salaries is broadly similar  
only 2 females achieved same salary as top 25% males  
majority of males earned more than the average female  
any other reasonable comment

S2 A bag contains 4 blue and 2 red counters. 2 counters are drawn at random without replacement.  
The random variable  $X$  is the number of blue counters drawn.  
[5] (a) Find the probability distribution for  $X$ . 4  
(b) Find  $E(X)$ . 2

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
S2		6	C	4.2.11/12	NC	05/22

- <sup>1</sup> ss know to find  $P(X=0)$  etc
- <sup>2</sup> pd process
- <sup>3</sup> pd process
- <sup>4</sup> pd process
- <sup>5</sup> ss choose correct form
- <sup>6</sup> pd process

- <sup>1</sup>  $\frac{1}{15}$
- <sup>2</sup>  $\frac{2}{6} \times \frac{4}{5}$
- <sup>3</sup>  $\dots + \frac{4}{6} \times \frac{2}{5} = \frac{8}{15}$
- <sup>4</sup>  $\frac{2}{5}$  4 marks
- <sup>5</sup>  $E(X) = \sum xp(x)$
- <sup>6</sup>  $\frac{4}{3}$  2 marks

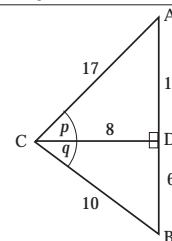
S3 A continuous random variable  $T$  has probability density function  $f(t) = \begin{cases} 5t^k & 0 < t < 1 \\ 0 & \text{otherwise} \end{cases}$ .  
[10] (a) Find the value of  $k$ . (b) Calculate  $P(0 < T < \frac{1}{2})$ . 4,3

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
S3		6	B	4.3.2	CN	05/65

- <sup>1</sup> ss know  $\int_0^1 f(t)dt = 1$
- <sup>2</sup> pd process
- <sup>3</sup> pd process
- <sup>4</sup> pd process
- <sup>5</sup> ss use  $\int_0^{\frac{1}{2}} 5t^k dt$
- <sup>6</sup> pd integrate
- <sup>7</sup> pd process limits

- <sup>1</sup>  $\int_0^1 f(t)dt = 1$
- <sup>2</sup>  $\int_0^1 5t^k dt = \frac{5}{k+1} t^{k+1}$
- <sup>3</sup>  $\frac{5}{k+1}$
- <sup>4</sup>  $k = 4$  4 marks
- <sup>5</sup>  $\int_0^{\frac{1}{2}} 5t^4 dt$
- <sup>6</sup>  $[t^5]_0^{\frac{1}{2}}$
- <sup>6</sup>  $\frac{1}{32}$  3 marks

2 Triangles ACD and BCD are right-angled at D with angles  $p$  and  $q$  and lengths as shown in the diagram.



- (a) Show that the exact value of  $\sin(p + q)$  is  $\frac{84}{85}$ .
- (b) Calculate the exact values of
- (i)  $\cos(p + q)$  (ii)  $\tan(p + q)$ .

4  
3

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
2	a	4	C	T9	CN	05/41
	b	3	C	T9	CN	

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> ic: interpret diagram
- <sup>2</sup> ic: interpret diagram
- <sup>3</sup> ss: expand  $\sin(A+B)$
- <sup>4</sup> pd: sub. and complete
- <sup>5</sup> ss: expand  $\cos(A+B)$
- <sup>6</sup> pd: sub. and complete
- <sup>7</sup> ic: use  $\tan(x) = \sin(x) / \cos(x)$

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $\cos(p) = \frac{8}{17}, \sin(p) = \frac{15}{17}$  [Note 1]
- <sup>2</sup>  $\cos(q) = \frac{8}{10}, \sin(q) = \frac{6}{10}$  **stated or implied by •4 when written in the same order as •3**
- <sup>3</sup>  $\sin(p)\cos(q) + \cos(p)\sin(q)$  **explicitly stated**
- <sup>4</sup>  $\frac{15}{17} \times \frac{8}{10} + \frac{8}{17} \times \frac{6}{10} = \& \text{ complete}$  **4 marks**
- <sup>5</sup>  $\cos(p)\cos(q) - \sin(p)\sin(q)$
- <sup>6</sup>  $-\frac{13}{85}$  or equivalent fraction
- <sup>7</sup>  $-\frac{84}{13}$  or equivalent fraction (eg  $-\frac{7140}{1105}$ ) **3 marks**

**Notes**

1 •<sup>1</sup> and •<sup>2</sup> may, if necessary, be awarded as follows

- <sup>1</sup>  $\sin(p) = \frac{15}{17}, \sin(q) = \frac{6}{10}$
- <sup>2</sup>  $\cos(p) = \frac{8}{17}, \cos(q) = \frac{8}{10}$

2 For •<sup>4</sup>

There has to be some working to show the completion.

eg  $\dots\dots\dots = \frac{120+48}{170} = \frac{168}{170} = \frac{84}{85}$

or

$\dots\dots\dots = \frac{60}{85} + \frac{24}{85} = \frac{84}{85}$

or

$\dots\dots\dots = \frac{12}{17} + \frac{24}{85} = \frac{84}{85}$

3 Calculating approx angles using  $\text{inv sin}$  and  $\text{inv cos}$  can gain no credit at any point.

4 Any attempt to use  $\sin(p + q) = \sin(p) + \sin(q)$  **loses •3 and •4.**

Any attempt to use  $\cos(p + q) = \cos(p) + \cos(q)$  **loses •5 and •6.**

**This second option must not be treated as a repeated error.**

**Alternative 1 (for marks 3 & 4)**

- <sup>3</sup>  $\frac{21}{\sin(p + q)} = \frac{10}{\frac{8}{17}}$
- <sup>4</sup>  $10 \sin(p + q) = \frac{168}{17}$  and complete

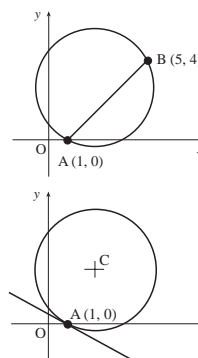
**Alternative 2 (for marks 5 & 6)**

- <sup>5</sup>  $\cos(p + q) = \frac{17^2 + 10^2 - 21^2}{2 \cdot 17 \cdot 10}$
- <sup>6</sup>  $-\frac{13}{85}$

**Alternative 3 (for marks 5 & 6)**

- <sup>5</sup>  $\cos^2(p + q) = 1 - \left(\frac{84}{85}\right)^2$
- <sup>6</sup>  $\cos(p + q) = -\frac{13}{85}$  with justification of the choice of negative sign  
e.g.  $(15 + 6)^2 (= 441) > 17^2 + 10^2 (= 389)$   
or using the cosine rule

- 3 (a) A chord joins the points A(1, 0) and B(5, 4) on the circle as shown in the diagram.  
Show that the equation of the perpendicular bisector of chord AB is  $x + y = 5$ .
- (b) The point C is the centre of this circle. The tangent at the point A on the circle has equation  $x + 3y = 1$ .  
Find the equation of the radius CA.
- (c) (i) Determine the coordinates of the point C.  
(ii) Find the equation of the circle.



4  
4  
4

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
3	a	4	C	G7	CN	05/44
	b	4	C	G15	CN	
	c	4	C	G10	CN	

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> ss: find perp. bisector
- <sup>2</sup> pd: calc. perp. gradient
- <sup>3</sup> ss: find approp. mid-point
- <sup>4</sup> ic: complete proof
- <sup>5</sup> ss: compare with  $y = mx + c$
- <sup>6</sup> ic: state gradient
- <sup>7</sup> ss: find gradient of radius
- <sup>8</sup> ic: state equation of line
- <sup>9</sup> ss: solve sim. equations
- <sup>10</sup> pd: solve sim. equations
- <sup>11</sup> ic: state equation of circle
- <sup>12</sup> pd: calculate radius

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $m_{AB} = 1$
- <sup>2</sup>  $m_{\perp} = -1$
- <sup>3</sup> midpoint = (3, 2)
- <sup>4</sup>  $y - 2 = -1(x - 3)$  *and* complete [Notes 1,2] 4 marks
- <sup>5</sup>  $y = -\frac{1}{3}x + \dots$  stated/implied by •6
- <sup>6</sup>  $m_{tgt} = -\frac{1}{3}$
- <sup>7</sup>  $m_{rad} = 3$  stated/implied by •8
- <sup>8</sup>  $y - 0 = 3(x - 1)$  [Note 3] 4 marks
- <sup>9</sup> use  $x + y = 5$  and  $y = 3x - 3$  [Notes 4,5]
- <sup>10</sup>  $x = 2, y = 3$
- <sup>11</sup>  $(x - 2)^2 + (y - 3)^2 = r^2$
- <sup>12</sup>  $r^2 = 10$  [Note 6] 4 marks

**Notes**

- 1 To gain •4 some evidence of completion needs to be shown  
eg  $y - 2 = -1(x - 3)$   
 $y - 2 = -x + 3$   
 $y + x = 5$
- 2 •4 is only available if an attempt has been made to find and use both a perpendicular gradient and a midpoint.
- 3 •8 is only available if an attempt has been made to find and use a perpendicular gradient.
- 4 At the •9, •10 stage  
Guessing (2,3) (from stepping) and checking it lies on perp. bisector of AB may be awarded •9 and •10  
Guessing (2,3) (with or without reason) and with no check gains **neither** •9 nor •10
- 5 Solving  $y = 3x - 3$  and  $x + 3y = 1$  leading to (1,0) will lose •9 and •10.
- 6 to gain •12 some evidence of use of the distance formula needs to be shown.
- 7 At the •11 and •12 stage  
Subsequent to a guess for the coordinates of C, •11 and •12 are only available if the guess is such that  $0 < x < 5$  and  $0 < y < 4$ .

**Alternative 1 [for •9 and •10]**

- <sup>9</sup> D=(3,6) where D is intersection of the perp. to AB through B and the circle.
- <sup>10</sup> C = midpoint of AD = (2,3)

**Common Error 1 [for •5 to •8]**

$$3y = -x + 1$$

$$m = -1$$

$$m_{rad} = 1$$

$$y - 0 = 1(x - 1)$$

•5 × •6 × •7 × *eased* •8 ×  $\sqrt{\quad}$   
*award 1 mark*

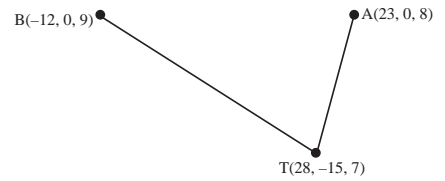
**Common Error 2 [for •5 to •8]**

$$x + 3y = 1 \text{ so } m = 3$$

$$y - 0 = 3(x - 1)$$

*award 0 marks*

4 The sketch shows the positions of Andrew(A), Bob (B) and Tracy(T) on three hill-tops.  
Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(-12, 0, 9) and T(28, -15, 7).  
In the dark, Andrew and Bob locate Tracy using heat-seeking beams.



- (a) Express the vectors  $\vec{TA}$  and  $\vec{TB}$  in component form. 2  
(b) Calculate the angle between these two beams. 5

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
4	a	2	C	G17	CN	05/55
	b	5	C	G28	Ca	

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> ic: state vector components
- <sup>2</sup> ic: state vector components
- <sup>3</sup> pd: find length of vector
- <sup>4</sup> pd: find length of vector
- <sup>5</sup> pd: find scalar product
- <sup>6</sup> ss: use scalar product
- <sup>7</sup> pd: evaluate angle

**Notes**

**In (a)**

- 1 For calculating  $\vec{AT}$  and  $\vec{BT}$  award 1 mark out of 2.
- 2 Treat column vectors written like  $(-40, 15, 2)$  as bad form.

**In (b)**

- 3 For candidates who do not attempt •7, the formula quoted at •6 must relate to the labelling in the question for •6 to be awarded.
- 4 Do not penalise premature rounding.

5 The use of  $\tan(A\hat{T}B) = \frac{\vec{TA} \cdot \vec{TB}}{|\vec{TA}| |\vec{TB}|}$  loses •6

6 The use of  $\cos(A\hat{T}B) = \frac{\vec{TA} \cdot \vec{TB}}{|\vec{TA}| |\vec{TB}|}$  means that only •5 and

•7 are available.

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $\vec{TA} = \begin{pmatrix} -5 \\ 15 \\ 1 \end{pmatrix}$
- <sup>2</sup>  $\vec{TB} = \begin{pmatrix} -40 \\ 15 \\ 2 \end{pmatrix}$  [Notes 1,2] 2 marks
- <sup>3</sup>  $|\vec{TA}| = \sqrt{251}$
- <sup>4</sup>  $|\vec{TB}| = \sqrt{1829}$
- <sup>5</sup>  $\vec{TA} \cdot \vec{TB} = 427$
- <sup>6</sup>  $\cos(A\hat{T}B) = \frac{\vec{TA} \cdot \vec{TB}}{|\vec{TA}| |\vec{TB}|}$  stated or implied by •7 [Note 3] 5 marks
- <sup>7</sup>  $A\hat{T}B = 50 \cdot 9^\circ$  OR  $0.889^c$  [Note 4]  
OR  $56.6$  grads

**Alternative 1 for •3 to •7 (Cosine Rule)**

- <sup>3</sup>  $|\vec{TA}| = \sqrt{251}$
- <sup>4</sup>  $|\vec{TB}| = \sqrt{1829}$
- <sup>5</sup>  $|\vec{AB}| = \sqrt{1226}$
- <sup>6</sup>  $\cos(A\hat{T}B) = \frac{1829 + 251 - 1226}{2 \cdot \sqrt{1829} \cdot \sqrt{251}}$  stated or implied by •7
- <sup>7</sup>  $A\hat{T}B = 50 \cdot 9^\circ$

**Common Error No.1**

- <sup>1</sup>  $\vec{TA} = t - a = \begin{pmatrix} 5 \\ -15 \\ -1 \end{pmatrix}$
- <sup>2</sup>  $\vec{TB} = t - b = \begin{pmatrix} 40 \\ -15 \\ -2 \end{pmatrix}$

award 1 mark

**Common Error No.2**

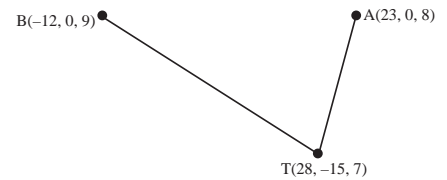
- <sup>1</sup>  $\vec{TA} = t + a = \begin{pmatrix} 51 \\ -15 \\ 15 \end{pmatrix}$
- <sup>2</sup>  $\vec{TB} = t + b = \begin{pmatrix} 16 \\ -15 \\ 16 \end{pmatrix}$

award 1 mark

Further common errors overleaf.



- 4 The sketch shows the positions of Andrew(A), Bob (B) and Tracy(T) on three hill-tops.  
Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(-12, 0, 9) and T(28, -15, 7).  
In the dark, Andrew and Bob locate Tracy using heat-seeking beams.



- (a) Express the vectors  $\vec{TA}$  and  $\vec{TB}$  in component form.  
(b) Calculate the angle between these two beams.

2  
5

**Common Error 1 : Finding angle BOA**

using  $\vec{OB} = \begin{pmatrix} -12 \\ 0 \\ 9 \end{pmatrix}$  and  $\vec{OA} = \begin{pmatrix} 23 \\ 0 \\ 8 \end{pmatrix}$

•  $|\vec{OB}| = \sqrt{225}$  and  $|\vec{OA}| = \sqrt{593}$

•  $\vec{OB} \cdot \vec{OA} = -204$

•  $\cos(\hat{BOA}) = \frac{\vec{OB} \cdot \vec{OA}}{|\vec{OB}| |\vec{OA}|}$

•  $\hat{BOA} = 124.0^\circ$  OR  $2.163^c$   
*award 1 mark per bullet*

**Common Error 2 : Finding angle BOT**

using  $\vec{OB} = \begin{pmatrix} -12 \\ 0 \\ 9 \end{pmatrix}$  and  $\vec{OT} = \begin{pmatrix} 28 \\ -15 \\ 7 \end{pmatrix}$

•  $|\vec{OB}| = \sqrt{225}$  and  $|\vec{OT}| = \sqrt{1058}$

•  $\vec{OB} \cdot \vec{OT} = -273$

•  $\left\langle \begin{aligned} \cos(\hat{BOT}) &= \frac{\vec{OB} \cdot \vec{OT}}{|\vec{OB}| |\vec{OT}|} \\ \hat{BOT} &= 124.0^\circ \text{ OR } 2.163^c \end{aligned} \right\rangle$

*award 1 mark per bullet*

**Common Error 3 : Finding angle AOT**

using  $\vec{OA} = \begin{pmatrix} 23 \\ 0 \\ 8 \end{pmatrix}$  and  $\vec{OT} = \begin{pmatrix} 28 \\ -15 \\ 7 \end{pmatrix}$

•  $|\vec{OA}| = \sqrt{593}$  and  $|\vec{OT}| = \sqrt{1058}$

•  $\vec{OA} \cdot \vec{OT} = 700$

•  $\left\langle \begin{aligned} \cos(\hat{AOT}) &= \frac{\vec{OA} \cdot \vec{OT}}{|\vec{OA}| |\vec{OT}|} \\ \hat{AOT} &= 27.9^\circ \text{ OR } 0.487^c \end{aligned} \right\rangle$

*award 1 mark per bullet*

**Common Error 4 : Finding angle ABT**

using  $\vec{BA} = \begin{pmatrix} 35 \\ 0 \\ -1 \end{pmatrix}$  and  $\vec{BT} = \begin{pmatrix} 40 \\ -15 \\ -2 \end{pmatrix}$

•  $|\vec{BA}| = \sqrt{1226}$  and  $|\vec{BT}| = \sqrt{1829}$

•  $\vec{BA} \cdot \vec{BT} = 1402$

•  $\left\langle \begin{aligned} \cos(\hat{ABT}) &= \frac{\vec{BA} \cdot \vec{BT}}{|\vec{BA}| |\vec{BT}|} \\ \hat{ABT} &= 20.6^\circ \text{ OR } 0.359^c \end{aligned} \right\rangle$

*award 1 mark per bullet*

**Common Error 5 : Finding angle BAT**

using  $\vec{AB} = \begin{pmatrix} -35 \\ 0 \\ 1 \end{pmatrix}$  and  $\vec{AT} = \begin{pmatrix} 5 \\ -15 \\ -1 \end{pmatrix}$

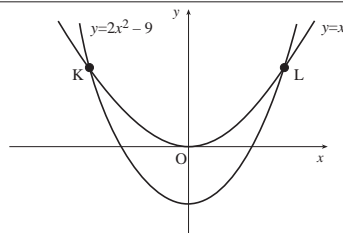
•  $|\vec{AB}| = \sqrt{1226}$  and  $|\vec{AT}| = \sqrt{251}$

•  $\vec{AB} \cdot \vec{AT} = -176$

•  $\left\langle \begin{aligned} \cos(\hat{BAT}) &= \frac{\vec{AB} \cdot \vec{AT}}{|\vec{AB}| |\vec{AT}|} \\ \hat{BAT} &= 108.5^\circ \text{ OR } 1.894^c \end{aligned} \right\rangle$

*award 1 mark per bullet*

- 5 The curves with equations  $y = x^2$  and  $y = 2x^2 - 9$  intersect at K and L as shown.  
Calculate the area enclosed between the curves.



8

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
5		8	C	C17	CN	05/49

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> ss: find intersection
- <sup>2</sup> pd: process quadratic equ.
- <sup>3</sup> ss: upper – lower
- <sup>4</sup> ic: interpret limits
- <sup>5</sup> pd: sub. & simplify Upper – Lower
- <sup>6</sup> pd: integrate
- <sup>7</sup> ic: substitute limits
- <sup>8</sup> pd: evaluate and complete

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $x^2 = 2x^2 - 9$
- <sup>2</sup>  $x = \pm 3$
- <sup>3</sup>  $\int \text{upper} - \text{lower}$  [Notes 3,4] stated or implied by •5
- <sup>4</sup> eg  $\int_0^3 \dots$
- <sup>5</sup>  $x^2 - 2x^2 + 9$
- <sup>6</sup>  $\left[-\frac{1}{3}x^3 + 9x\right]_0^3$
- <sup>7</sup>  $\left(-\frac{1}{3} \times 3^3 + 9 \times 3\right) - 0$
- <sup>8</sup>  $2 \times 18 = 36$  [Note 3] 8 marks

### Notes

- 1 There is no penalty for working with  $\frac{1}{3}x^3 - \frac{2}{3}x^3 + 9x$  or even  $\frac{1}{3}x^3 - \left(\frac{2}{3}x^3 - 9x\right)$  but in the latter case, the minus signs need to be dealt with correctly at some point for •5 to be awarded.
- 2 Candidates who attempt to find a solution using a graphics calculator earn no marks. The only acceptable solution is via calculus.
- 3 •3 is lost for subtracting the wrong way round and subsequently •8 may be lost for such statements as

–36  
–36 square units  
–36 = 36  
–36 so ignore the –ve  
–36 = 36 square units

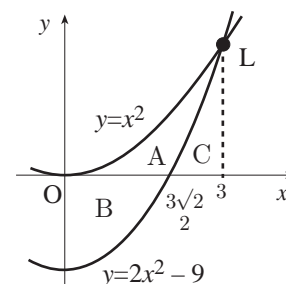
•8 may be gained for statements such as  
–36 so the area = 36

- 4  $\int_3^{-3} (\text{lower} - \text{upper})$  or  $\int_3^0 (\text{lower} - \text{upper})$  are technically correct and hence all 8 marks are available.
- 5 For  $\int_K^L (\text{upper} - \text{lower})$ , •3, •5, •6 and •7 are available
- 6 Differentiation loses •6, •7 and •8.
- 7 Using  $x^2 + 2x^2 - 9$  and  $\int_{-3}^3 (3x^2 - 9) dx$  leading to zero can only gain •4 and •6 from the last 6 marks.
- 8 Candidates may attempt to split the area up. In Alt.2, for candidates who treat “C” as a triangle, the last three marks are not available.

### Alternative 1 for •4 to •8

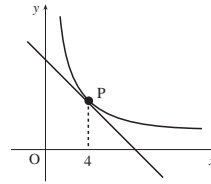
- <sup>4</sup> eg  $\int_{-3}^3 \dots$
- <sup>5</sup>  $x^2 - 2x^2 + 9$
- <sup>6</sup>  $\left[-\frac{1}{3}x^3 + 9x\right]_{-3}^3$
- <sup>7</sup>  $\left(-\frac{1}{3} \times 3^3 + 9 \times 3\right) - \left(-\frac{1}{3} \times (-3)^3 + 9 \times (-3)\right)$
- <sup>8</sup> 36

### Alternative 2 for •3 to •8



- <sup>3</sup>  $x = \frac{3}{2}\sqrt{2}$
- <sup>4</sup>  $\int_0^{\frac{3}{2}\sqrt{2}} (9 - 2x^2) dx$  leading to  $B = 9\sqrt{2}$  (12.7)
- <sup>5</sup>  $\int_0^3 (x^2) dx$  leading to  $A + C = 9$
- <sup>6</sup>  $\int_{\frac{3}{2}\sqrt{2}}^3 (2x^2 - 9) dx$  leading to  $C = 9\sqrt{2} - 9$  (3.7) [Note 8]
- <sup>7</sup>  $A = 18 - 9\sqrt{2}$  (5.3)
- <sup>8</sup> Total area = 36

6 The diagram shows the graph of  $y = \frac{24}{\sqrt{x}}$ ,  $x > 0$ .



Find the equation of the tangent at P, where  $x = 4$ .

6

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
6		6	B	C5, C3	CN	05/43

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> ss : know to differentiate
- <sup>2</sup> ic : express in st. form
- <sup>3</sup> pd : differentiate –ve fractional index
- <sup>4</sup> pd : evaluate –ve fractional index
- <sup>5</sup> pd : evaluate  $y$ -coord
- <sup>6</sup> ic : state equ of tangent

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $\frac{dy}{dx} = \dots$
- <sup>2</sup>  $y = 24x^{-\frac{1}{2}}$
- <sup>3</sup>  $\frac{dy}{dx} = -12x^{-\frac{3}{2}}$
- <sup>4</sup>  $\frac{dy}{dx}_{x=4} = -\frac{3}{2}$
- <sup>5</sup>  $y_{x=4} = 12$
- <sup>6</sup>  $y - 12 = -\frac{3}{2}(x - 4)$  [Notes 1,2,3] **6 marks**

nr  $[2y + 3x = 36]$

nr = not required

#### Notes

- 1 •<sup>4</sup> and •<sup>6</sup> are only available if an attempt to find the gradient is based on differential calculus.
- 2 •<sup>6</sup> is not available to candidates who find and use a perpendicular gradient.
- 3 •<sup>6</sup> is only available for a numerical value of  $m$ .

#### Common Error 1

- <sup>1</sup>  $\frac{dy}{dx} = \dots$
- <sup>2</sup>  $y = 24x^{-\frac{1}{2}}$
- <sup>3</sup>  $\frac{dy}{dx} = \frac{24x^{\frac{1}{2}}}{\frac{1}{2}}$
- <sup>4</sup>  $\frac{dy}{dx}_{x=4} = 96$
- <sup>5</sup>  $y_{x=4} = 12$
- <sup>6</sup>  $y - 12 = 96(x - 4)$

- 1 ✓
- 2 ✓
- 3 ×
- 4 × eased
- 5 ✓
- 6 × ✓

award 4 marks

#### Common Error 2

- <sup>1</sup>
- <sup>2</sup>  $y = 24x^{-\frac{1}{2}}$
- <sup>3</sup>  $\int 24x^{-\frac{1}{2}} dx = \frac{24x^{\frac{1}{2}}}{\frac{1}{2}} + c$
- <sup>4</sup> gradient = 96
- <sup>5</sup>  $y_{x=4} = 12$
- <sup>6</sup>  $y - 12 = 96(x - 4)$

- 1 ×
- 2 ✓
- 3 ×
- 4 × Note 1
- 5 ✓
- 6 × Note 1

award 2 marks

7 Solve the equation  $\log_4(5-x) - \log_4(3-x) = 2$ ,  $x < 3$ .

4

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
7		4	A	A7	CN	0525

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> ss: use the log laws
- <sup>2</sup> ss: know to convert from log to expo
- <sup>3</sup> pd: process conversion
- <sup>4</sup> pd: find valid solution

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $\log_4\left(\frac{5-x}{3-x}\right)$
- <sup>2</sup> use  $\log_a(b) = c \Leftrightarrow b = a^c$  **stated or implied by •3**
- <sup>3</sup>  $\frac{5-x}{3-x} = 4^2$  **See Cave**
- <sup>4</sup>  $x = \frac{43}{15}$  **4 marks**

### Notes

1 For •4

Accept answer as a decimal.

#### Common Error No.1

•1  $\checkmark \log_4\left(\frac{5-x}{3-x}\right) = \log_4(8)$

•2  $\times$

•3  $\times$

$$\frac{5-x}{3-x} = 8$$

•4  $\times \checkmark x = \frac{19}{7}$

*award 2 marks*

#### Common Error No.2

•1  $\checkmark \log_4\left(\frac{5-x}{3-x}\right) = 2$

•2  $\times \frac{5-x}{4^{3-x}} = 2$

•3  $\times$

$$\frac{5-x}{3-x} = \frac{1}{2}$$

•4  $\times \checkmark x = 7$  which is not a valid sol.

*award 2 marks*

#### Common Error No.3

•1  $\checkmark \log_4\left(\frac{5-x}{3-x}\right) = 2$

•2  $\times \log_4\left(\frac{5-x}{3-x}\right) = \log_4 2$

•3  $\times \frac{5-x}{3-x} = 2$

•4  $\times \checkmark x = 1$ .

*award 2 marks*

### Alternative 1

•<sup>1</sup>  $\log_4\left(\frac{5-x}{3-x}\right)$

•<sup>2</sup>  $2\log_4 4$

•<sup>3</sup>  $\left(\frac{5-x}{3-x}\right) = 4^2$

•<sup>4</sup>  $x = \frac{43}{15}$

**stated or implied by •3**

### Cave

$$\log_4\left(\frac{5-x}{3-x}\right)$$

$$\frac{5-x}{3-x} = 16$$

leading to

$$x = \frac{43}{15}$$

*award 4 marks*

**BUT**

$$\log_4\left(\frac{5-x}{3-x}\right)$$

$$\frac{5-x}{3-x} = 2^4$$

leading to

$$x = \frac{43}{15}$$

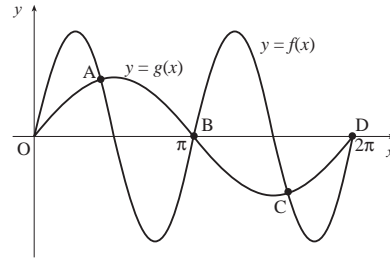
•<sup>1</sup>  $\checkmark$ , •<sup>2</sup>  $\times$ , •<sup>3</sup>  $\times$ , •<sup>4</sup>  $\times \checkmark$

*award 2 marks*

- 8 Two functions,  $f$  and  $g$ , are defined by  
 $f(x) = k\sin(2x)$  and  $g(x) = \sin(x)$  where  $k > 1$ .

The diagram shows the graphs of

$y = f(x)$  and  $y = g(x)$  intersecting at O, A, B, C and D. Show that, at A and C,  $\cos(x) = \frac{1}{2k}$ .



5

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
8		5	A	T10	CN	05/47

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> ss: equate for intersection
- <sup>2</sup> ss: use double angle formula
- <sup>3</sup> pd: factorise
- <sup>4</sup> pd: process two solutions
- <sup>5</sup> ic: complete proof

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $k \sin(2x) = \sin(x)$  [Note 1]
- <sup>2</sup>  $k \times 2 \sin(x) \cos(x)$
- <sup>3</sup>  $\sin(x)(2k \cos(x) - 1) = 0$
- <sup>4</sup>  $\sin(x) = 0$   
**and**  $\cos(x) = \frac{1}{2k}$
- <sup>5</sup>  $\sin(x) = 0 \Rightarrow x = 0, \pi, 2\pi$   
 i.e. at (O), B and D [Note 2]  
**and**  $\cos(x) = \frac{1}{2k}$  is for A and C. 5 marks

#### Notes

- 1 Only •1 is available for candidates who substitute a numerical value for  $k$  at the start.
- 2 •5 is only available if a suitable comment regarding points (O), B and D is made.
- 3 If all the terms are transposed to one side, then an “=0” needs to appear at least once.
- 4 For Alternative 3  
 •4 and •5 are not available unless •3 has been awarded.

#### Common Error 1

- <sup>1</sup> ✓  $k \sin(2x) = \sin(x)$
- <sup>2</sup> ✓  $k \times 2 \sin(x) \cos(x) - \sin(x) = 0$
- <sup>3</sup> ✓  $\sin(x)(2k \cos(x) - 1)$
- <sup>4</sup> ×  $2k \cos(x) - 1 = 0$
- <sup>5</sup> ×  $\cos(x) = \frac{1}{2k}$  at A and C.

award 3 marks

#### Common Error 2

- <sup>1</sup> ✓  $k \sin(2x) = \sin(x)$
- <sup>2</sup> ✓  $k \times 2 \sin(x) \cos(x) = \sin(x)$
- <sup>3</sup> ×  $k \times 2 \cos(x) = 1$
- <sup>4</sup> ×
- <sup>5</sup> ×  $\cos(x) = \frac{1}{2k}$  at A and C.

award 2 marks

#### Alternative 1 for •4 and •5

- <sup>4</sup> at (O), B and D,  $\sin(x) = 0$
- <sup>5</sup> so at A and C,  $2k \cos(x) - 1 = 0$   
 $\Rightarrow \cos(x) = \frac{1}{2k}$ .

#### Alternative 2 for •4 and •5

- <sup>4</sup> at A and C,  $\sin(x) \neq 0$
- <sup>5</sup> so at A and C,  $2k \cos(x) - 1 = 0$   
 $\Rightarrow \cos(x) = \frac{1}{2k}$ .

#### Alternative 3 for •1 to •5

- <sup>1</sup>  $k \sin(2x) = \sin(x)$
- <sup>2</sup>  $k \times 2 \sin(x) \cos(x) = \sin(x)$
- <sup>3</sup> at A and C,  $\sin(x) \neq 0$
- <sup>4</sup> so at A and C,  $2k \cos(x) = 1$
- <sup>5</sup>  $\cos(x) = \frac{1}{2k}$

- 9 The value  $V$  (in £ million) of a cruise ship  $t$  years after launch is given by the formula

$$V = 252e^{-0.06335t}.$$

- (a) What was its value when launched? 1  
 (b) The owners decide to sell the ship once its value falls below £20 million. After how many years will it be sold? 4

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
9	a	1	B	A34	CN	05/76
	b	4	A	A34	Ca	

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> pd: evaluate at  $t = 0$
- <sup>2</sup> ic: substitute  $V = 20$
- <sup>3</sup> pd: process
- <sup>4</sup> ic: expo to log conversion
- <sup>5</sup> pd: solve a logarithmic equation

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $V_{t=0} = 252$  (£m) 1 mark
- <sup>2</sup>  $252e^{-0.06335t} = 20$
- <sup>3</sup>  $e^{-0.06335t} = \frac{20}{252}$
- <sup>4</sup>  $-0.06335t = \ln\left(\frac{20}{252}\right)$
- <sup>5</sup>  $t = 40$  [Note 1] 4 marks

### Notes

#### in (b)

- For •<sup>5</sup> accept any correct answer which rounds to 40.
- An answer obtained by trial and improvement which rounds to 40 may be awarded a max. of 1 mark (out of 4) **but only** if they have checked 39 as well.
- In following through from an error, •<sup>5</sup> is only available for a positive answer.

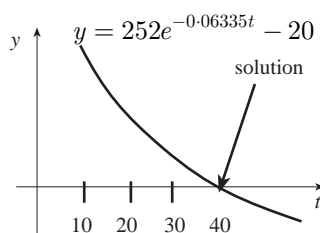
#### Common Error 1

- <sup>2</sup> ✓  $\log(252e^{-0.06335t}) = \log 20$
- <sup>3</sup> ×  $-0.06335t \log 252 = \log 20$
- <sup>4</sup> ×  $-0.06335t = \frac{\log 20}{\log 252}$
- <sup>5</sup> ×  $t = -8.55$

**award 1 mark**

#### Solution via graphics calculator

- <sup>2</sup>  $252e^{-0.06335t} = 20$
- <sup>3</sup> choose to graph  $y = 252e^{-0.06335t} - 20$
- <sup>4</sup> a sketch [see below]
- <sup>5</sup>  $t = 40$



### Note

You could also graph, for example,  $y = 252e^{-0.06335t}$  and  $y = 20$

#### Alternative 1 for (b) (takings logs of both sides)

- <sup>2</sup>  $252e^{-0.06335t} = 20$
- <sup>3</sup>  $e^{-0.06335t} = \frac{20}{252}$
- <sup>4</sup>  $-0.06335t \log_k(e) = \log_k\left(\frac{20}{252}\right)$   
where  $k = e$  or  $k = 10$
- <sup>5</sup>  $t = 40$  [Note 1]

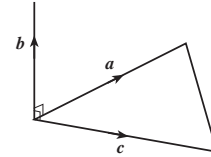
#### Alternative 2

- <sup>2</sup>  $252e^{-0.06335t} = 20$
- <sup>3</sup>  $\log 252 - 0.06335t \log e = \log 20$
- <sup>4</sup>  $5.53 - 0.06335t = 2.99$
- <sup>5</sup>  $t = 40$

#### Alternative 3

- <sup>2</sup>  $252e^{-0.06335t} = 20$
- <sup>3</sup>  $\ln 252 + \ln e^{-0.06335t} = \ln 20$
- <sup>4</sup>  $-0.06335t \ln e = \ln 20 - \ln 252$
- <sup>5</sup>  $t = 40$

10 Vectors  $\mathbf{a}$  and  $\mathbf{c}$  are represented by two sides of an equilateral triangle with sides of length 3 units, as shown in the diagram. Vector  $\mathbf{b}$  is 2 units long and  $\mathbf{b}$  is perpendicular to both  $\mathbf{a}$  and  $\mathbf{c}$ . Evaluate the scalar product  $\mathbf{a} \cdot (\mathbf{a} + \mathbf{b} + \mathbf{c})$ .



4

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
10		4	A	G29	CN	05/31

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> ss: use distributive law
- <sup>2</sup> pd: process scalar product
- <sup>3</sup> pd: process scalar product
- <sup>4</sup> pd: process scalar product & complete

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $\mathbf{a} \cdot \mathbf{a} + \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$  see CAVE [Notes 1,2]
- <sup>2</sup>  $\mathbf{a} \cdot \mathbf{a} = 9$
- <sup>3</sup>  $\mathbf{a} \cdot \mathbf{c} = \frac{9}{2}$  [Note 3]
- <sup>4</sup>  $\mathbf{a} \cdot \mathbf{b} = 0$  and a total of  $13\frac{1}{2}$  4 marks

**Notes**

- 1 Treat  $\underline{a.a}$  written as  $a^2$  as bad form.
- 2 Treat  $\underline{a.b}$  written as  $ab$  as an error unless it is subsequently evaluated as a scalar product. Similarly for  $\underline{a.c}$ .
- 3 Using  $\underline{p.q} = |p||q|\sin\theta$  consistently loses 1 mark. (ie max. available is 3)
- 4 When attaching the components

$$\mathbf{c} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}, \mathbf{a} = \begin{pmatrix} \frac{3}{2} \\ \frac{3\sqrt{3}}{2} \\ 0 \end{pmatrix}, \text{ all marks are available.}$$

When attaching the components

$$\mathbf{c} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}, \mathbf{a} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}, \text{ only } \bullet 1 \text{ is available.}$$

**CAVE**

$\mathbf{a} \cdot (\mathbf{a} + \mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{a} + \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$   
 followed by  
 $\mathbf{a} \cdot \mathbf{a} = 9$   
 earns  $\bullet 1$  and  $\bullet 2$ .

but

$\mathbf{a} \cdot (\mathbf{a} + \mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{a} + \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$   
 followed by  
 $\mathbf{a} \cdot \mathbf{a} = 9, \mathbf{a} \cdot \mathbf{c} = 9, \mathbf{a} \cdot \mathbf{b} = 6$   
 earns  $\bullet 1$  only.

- 11 (a) Show that  $x = -1$  is a solution of the cubic equation  $x^3 + px^2 + px + 1 = 0$ . 1  
 (b) Hence find the range of values of  $p$  for which all the roots of the cubic equation are real. 7

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
11	a	1	C	A21	CN	05/54
	b	7	A	A22	CN	

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> pd: evaluate the function at  $x = -1$
- <sup>2</sup> ss: strategy for finding other factors
- <sup>3</sup> ic: quadratic factor
- <sup>4</sup> ss: strategy for real roots
- <sup>5</sup> ic: substitute
- <sup>6</sup> pd: process
- <sup>7</sup> ss: starts to solve inequation
- <sup>8</sup> ic: complete

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $f(-1) = -1 + p - p + 1 = 0$  1 mark
- <sup>2</sup>  $-1 \begin{array}{cccc} 1 & p & p & 1 \\ & -1 & 1-p & -1 \\ 1 & p-1 & 1 & 0 \end{array}$
- <sup>3</sup>  $x^2 + (p-1)x + 1 = 0$  [Note 2]
- <sup>4</sup> " $b^2 - 4ac$ " and " $\geq 0$ " [Notes 3,4]
- <sup>5</sup>  $(p-1)^2 - 4$
- <sup>6</sup>  $(p-3)(p+1)$
- <sup>7</sup>  $p = 3, p = -1$
- <sup>8</sup>  $p \leq -1, p \geq 3$  [Note 6] 7 marks

**Notes**

- 1 For alternative method 1, •<sup>2</sup>  
 •<sup>2</sup> (as is •<sup>3</sup> also) is for interpreting the result of a synthetic division.  
 Candidates must show some acknowledgement of the result of the synthetic division. Although a statement w.r.t. the zero is preferable, accept something as simple as "underlining" the zero.
- 2 Treat "= 0" missing at •<sup>3</sup> as Bad Form
- 3 •<sup>4</sup> is only available as a consequence of obtaining a quadratic factor from a division of the cubic.
- 4 Using  $b^2 - 4ac > 0$  loses •<sup>4</sup>  
 An "≥" must appear at least once somewhere between •<sup>4</sup> and •<sup>6</sup>
- 5 Where errors occur at the •<sup>3</sup>/<sup>5</sup> stages, then •<sup>6</sup>,•<sup>7</sup>,•<sup>8</sup> are still available for solving a '3-term' quadratic inequation.
- 6 Evidence for •<sup>8</sup> may be a table of values or a sketch
- 7 For candidates who start with  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ , all marks are available (subject to working being equivalent to the Primary Method).
- 8 Wrong discriminant:  
 Using  $b^2 + 4ac$  only •<sup>5</sup> (out of the last 5 marks) is available.  
 Any other expression masquerading as the discriminant loses all of the last 5 marks.

Alternative method 1 for marks 1,2 (starting with synth. division)

- <sup>1</sup>  $-1 \begin{array}{cccc} 1 & p & p & 1 \\ & -1 & 1-p & -1 \\ 1 & p-1 & 1 & 0 \end{array}$
- <sup>2</sup>  $f(-1)=0$  [Note 1]
- etc

**Marks should still be recorded as out of 1 and 7**

Alternative method 2 for marks 1,2 (quad. factor obtained by inspection)

- <sup>1</sup>  $f(-1) = -1 + p - p + 1 = 0$
- <sup>2</sup>  $f(x) = (x+1)(x^2 \dots\dots\dots)$
- etc

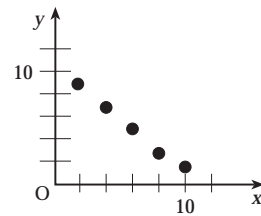
Common Error 1 (marks 5 to 8)

$$\begin{aligned} (p-1)^2 - 4 &\geq 0 \\ (p-1)^2 &\geq 4 \\ p-1 &\geq 2 \\ p &\geq 3 \end{aligned}$$

*award 2 marks out of last 4*



[4] The scatter diagram shows 5 pairs of data values for  $x$  and  $y$  where  
 $\Sigma x = 30$ ,  $\Sigma y = 26$ ,  $\Sigma x^2 = 220$ ,  $\Sigma y^2 = 168$  and  $\Sigma xy = 120$ .



- (a) Find the equation of the regression line.  
 (b) Estimate the value of  $y$  when  $x = 5$ .

4  
1

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
S1	a	4	C	4.4.2	Ca	05/76
	b	1	C	4.4.2	CN	

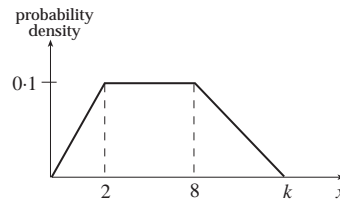
The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> pd: calculate  $S_{xy}$
- <sup>2</sup> pd: calculate  $S_{xx}$
- <sup>3</sup> pd: calculate  $b$
- <sup>4</sup> pd: calculate  $a$  & state equ.
- <sup>5</sup> ic: use equ. of regression line

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $S_{xy} = -36$
- <sup>2</sup>  $S_{xx} = 40$
- <sup>3</sup>  $b = -0.9$
- <sup>4</sup>  $a = 10.6$  and  $y = 10.6 - 0.9x$  4 marks
- <sup>5</sup>  $y_{x=5} = 6.1$  1 mark

[7] The diagram represents the probability density function for a continuous random variable  $X$ .



- (a) Find the value of  $k$ .  
 (b) Find the median.

3  
2

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
S4	a	3	A	4.3.1	CN	05/83
	b	2	A	4.3.5	CN	

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- <sup>1</sup> ss: state total area = 1
- <sup>2</sup> ic: find expression for total area
- <sup>3</sup> pd: process
- <sup>4</sup> ss: know total area = 0.5
- <sup>5</sup> pd: process

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $area = 1$
- <sup>2</sup>  $0.1 + 0.6 + \frac{1}{2}(k - 8) \times 0.1$
- <sup>3</sup>  $k = 14$  3 marks
- <sup>4</sup>  $0.1 + (m - 2) \times 0.1 = \frac{1}{2}$
- <sup>5</sup>  $m = 6$  2 marks

[9]	(a)	Explain briefly the difference between sample standard deviation and range as measures of spread.	<div style="border: 1px solid black; border-radius: 15px; padding: 5px; width: fit-content; margin: auto;"> <math>\bar{x} = 2.325</math>  <math>S_x = 0.573883355</math>  <math>\sigma_x = 0.559352304</math>  <math>\Sigma x = 46.5</math>  <math>\Sigma x^2 = 114.37</math>  <math>n = 20</math>  <math>x_{\min} = 1.2</math>  <math>x_{\max} = 3.2</math> </div>	1
	(b)	In statistics mode, a calculator shows the summary statistics for a certain data set.  One data value, 1.2, is shown to be erroneous and is deleted.  Calculate the sample standard deviation of the new data set of 19 values correct to 3 decimal places.		4

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
S3	a	1	B	4.2.11/12	CN	05/79
	b	4	B	4.1.1	Ca	

<p>The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME</p>
<ul style="list-style-type: none"> <li>•<sup>1</sup> ic: explanation</li> <li>•<sup>2</sup> pd: find new <math>\Sigma x</math></li> <li>•<sup>3</sup> pd: find new <math>\Sigma x^2</math></li> <li>•<sup>4</sup> ss: use formula for <math>S_x</math></li> <li>•<sup>5</sup> pd: process</li> </ul>

<p><b>Primary Method : Give 1 mark for each •</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> SD is a measure of spread about mean whereas <math>(x_{\max} - x_{\min})</math> is a measure of range. <b>1 mark</b></li> <li>•<sup>2</sup> <math>\Sigma x = 45.3</math></li> <li>•<sup>3</sup> <math>\Sigma x^2 = 112.93</math></li> <li>•<sup>4</sup> <math>S = \sqrt{\frac{1}{18} \left( 112.93 - \frac{45.3^2}{19} \right)}</math></li> <li>•<sup>5</sup> <math>0.523</math></li> </ul> <p style="text-align: right;"><b>4 marks</b></p>
---

[10]	(a)	A large organisation decides to run a mini-lottery for charity.	
		<ul style="list-style-type: none"> <li>• Each participant selects any three different numbers from 1 to 20 inclusive.</li> <li>• Every Friday the three winning numbers are drawn at random from the 20.</li> <li>• Each participant with these winning numbers shares the jackpot.</li> </ul>	
	(a)	Find the number of possible combinations and hence find the probability of a particular combination winning a share of the jackpot.	2
	(b)	Find the probability that someone chooses the winning combination exactly twice within 3 successive weeks.	3

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
S4	a	2	B	4.2.5, 4.2.3	Ca	05/78
	b	3	A	4.2.7	Ca	

<p>The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME</p>
<ul style="list-style-type: none"> <li>•<sup>1</sup> ss: find combination</li> <li>•<sup>2</sup> pd: calculate probability</li> <li>•<sup>3</sup> ic: interpret p(win)</li> <li>•<sup>4</sup> ss: find combination</li> <li>•<sup>5</sup> pd: process</li> </ul>

<p><b>Primary Method : Give 1 mark for each •</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> No. of outcomes = <math>\binom{20}{3}</math></li> <li>•<sup>2</sup> <math>prob = \frac{1}{\binom{20}{3}} = \frac{1}{1140}</math> <span style="float: right;"><b>2 marks</b></span></li> <li>•<sup>3</sup> <math>p(L) = \frac{1139}{1140}</math></li> <li>•<sup>4</sup> <math>p(2 \text{ wins in } 3)</math>  <math>= 3 \times \left(\frac{1}{1140}\right)^2 \times \left(\frac{1139}{1140}\right)</math></li> <li>•<sup>5</sup> <math>2.306 \times 10^{-6}</math> <span style="float: right;"><b>3 marks</b></span></li> </ul>
---